Cosmic Strings
from
Supersymmetric Flat Directions

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Motivation

- New $U(1)$ gauge symmetries arise in many new physics models:
  - grand unified theories
  - $D$-brane constructions
  - superstring theory compactifications

- Supersymmetry is motivated by:
  - the gauge hierarchy problem
  - grand unification
  - cosmological observations (dark matter, baryon asymmetry, . . .)
  - superstring theory

- What are the cosmological consequences if these possibilities are realized in nature?

  $\rightarrow$ supersymmetric cosmic strings
Ingredients:
Cosmic Strings
and
Supersymmetry
Cosmic Strings

- Cosmic strings are non-trivial field configurations that can arise in theories containing scalar fields. [Nielsen+Olesen '73]

- Cosmic strings can be formed when a $U(1)$ (gauge) symmetry is spontaneously broken in the early universe.

- Some cosmic string signatures:
  - large-scale structure formation
  - gravitational lensing
  - gravity waves

- Cosmic superstrings can also arise from superstring theory. [Jones,Stoica,Tye '02; Dvali+Vilekin '03; Copeland,Myers,Polchinski '03]

$(p, q) \Rightarrow p$ fundamental $F$ strings and $q$ $D1$ branes
A Simple Cosmic String - Part 1

- **Abelian Higgs** model with a complex scalar $\varphi$, and a $U(1)$ gauge field $A_\mu$:

\[
\mathcal{L} \supset |(\partial_\mu - igA_\mu) \varphi|^2 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{\lambda}{4} (|\varphi|^2 - v^2)^2
\]

- Symmetry breaking vacuum: $\langle |\varphi| \rangle = v, \ A_\mu = 0$.

- This theory also has cosmic string configurations labelled by $N \in \mathbb{Z}^>$:

\[
\varphi(r, \phi, z) = v f(r) e^{iN\phi}, \quad \text{with} \quad f(r) \to \begin{cases} 1 & ; \ r \to \infty \\ 0 & ; \ r \to 0 \end{cases}
\]

\[
A_\phi(r, \phi, z) = \frac{N}{gr} a(r), \quad \text{with} \quad a(r) \to \begin{cases} 1 & ; \ r \to \infty \\ r^2 & ; \ r \to 0 \end{cases}
\]

- This configuration has finite energy per length for nice $f(r)$ and $a(r)$. → cosmic string with winding number $N$. 
\[ N = 1 : \quad \varphi = v f(r) e^{i\phi} \quad , \quad A_\mu \rightarrow A_\phi(r) \]
A Simple Cosmic String - Part 2

- This cosmic string configuration is stable on account of topology,

\[ \pi_1(S^1) = \mathbb{Z} \]

An infinite energy cost is required to change the winding number \( N \).

- The properties of the string are set by the scale of spontaneous symmetry breaking \( v = \langle |\varphi| \rangle \):

  \[
  \begin{align*}
  \text{String Width} & : \ w \approx v^{-1} \\
  \text{String Tension} & : \ \mu \approx v^2
  \end{align*}
  \]

- Usually only the \( N = 1 \) mode is stable.

- This example is typical of ordinary cosmic strings.
Supersymmetry (SUSY)

- Supersymmetry is a well-motivated possibility for new physics.

- Quantum corrections can destabilize the gauge hierarchy:
  \[ M_W \sim 100 \text{ GeV} \ll M_{Pl} \sim 10^{18} \text{ GeV} \]
  With SUSY, the dangerous quantum corrections cancel out.

- This feature makes SUSY theories a natural setting for the scalar fields that make up cosmic strings.

- The cancellations due to SUSY can also give rise to directions in the scalar potential that are extremely flat.
A Simple SUSY Flat Direction

- Sample SUSY flat-direction potential:

\[ V_{\text{eff}}(\varphi) = -m^2|\varphi|^2 + \frac{\lambda}{M^{2n}}|\varphi|^{4+2n}, \]

with \( m \ll M, \ n > 0 \).

- Supersymmetry allows for \( m \ll M \).

- The vacuum value of \(|\varphi|\) is

\[ v := \langle |\varphi| \rangle \simeq (mM^n)^{1/(n+1)}. \]

Note that \( m \ll v \ll M \).

We have in mind \( m \sim 10^3 \text{ GeV}, \ M \sim M_{\text{Pl}} \simeq 2.4 \times 10^{18} \text{ GeV} \).
Flat-Direction Strings
String Solutions

- We look for classical field solutions of the form

\[
\varphi(r, \phi) = v f(r) e^{iN\phi}, \\
A_\phi(r) = \frac{N}{gr} \tilde{a}(r),
\]

with the boundary conditions

\[
f(r), \tilde{a}(r) \to 1 \text{ as } r \to \infty, \\
f(r), \tilde{a}(r) \to 0 \text{ as } r \to 0.
\]

- \(N\) is a positive integer.

- We solve the equations of motion approximately and refine them using variational methods.
• The profile $\tilde{a}(r)$ has a width of $v^{-1}$.

• The profile $f(r)$ has width $m^{-1}$.

$\rightarrow$ corresponds to the flat direction
String Tensions

- Tension $= \mu = \text{energy per unit length}$
- Flat direction strings have tensions $\mu \sim v^2$ (like ordinary strings).
- To a good approximation,
  \[ \mu_1 \sim \frac{4\pi^2 v^2}{\log(v^2/m^2)}. \]
• The tension $\mu_N$ increases very slowly with the winding number $N$:

\[ \mu_N \simeq \mu_1 \left[ 1 + \frac{3}{\ln(v^2/m^2)} \ln N \right]. \]

$\Rightarrow \mu_{N+M} < \mu_N + \mu_M$

$\Rightarrow$ higher ($N > 1$) winding modes are energetically stable.
String Interactions
String Interactions: Intercommutation

- When a pair of ordinary strings intersect they can:
  1. pass through each other
  2. intercommute (reconnect)

- Flat strings also intercommute.
String Interactions: Zippering

- Flat-direction strings have a qualitatively new interaction mode because they have stable higher winding states.
- Two $N = 1$ strings can form a new segment with $N = 2$:

$$\alpha$$

- More generally, topology allows $N + M \rightarrow |N \pm M|$.
- Cosmic superstrings are also able to form zippers.
• $1 + 1 \rightarrow 2$

• Zippering is only kinematically allowed for $\mu_2 < 2 \mu_1$.

• Initial velocities $\nu \approx 0.6$ are typical in the early universe.

• For ordinary strings, $\mu_2 \approx 2 \mu_1$, and zippering is unlikely.
• $N + M \rightarrow |N \pm M|$ is also possible.

• (Intercommutation corresponds to $N + N \rightarrow 0$.)

• We assume that zippering occurs whenever it is kinematically allowed.
Comparison with Ordinary Strings

• Ordinary strings depend on a single dimensionful quantity $v$.
  
  – String Tension: $\mu \sim v^2$
  
  – String Width: $w \sim v^{-1}$
  
  – $\mu_2 \simeq 2 \mu_1$, and zippering is unlikely.

• Flat strings depend on two dimensionful quantities, $m \ll v$.
  
  – String Tension: $\mu \sim v^2$
  
  – String Width: $w \sim m^{-1} \gg v^{-1}$
  
  – $\mu_2 < 2 \mu_1$, and zippering is often possible.

• Cosmic superstrings are also able to zipper.
String Networks
in the
Early Universe
Cosmic String Scaling

- Scale factor: \( a(t) \propto \frac{1}{T(t)} \).

- The energy density of non-interacting strings scales as
  \[
  \rho_{\text{string}} \propto a^{-2}(t).
  \]

- This is a problem?
  \[
  \rho_{\text{matter}} \propto a^{-3}(t),
  \rho_{\text{radiation}} \propto a^{-4}(t).
  \]

- However, strings form loops by intercommutation, which decay away.

- Cosmic strings track the background matter or radiation density
  \[
  \rho_{\text{string}} \simeq G\mu (\rho_{\text{matter}} + \rho_{\text{radiation}})
  \]

  → cosmic string scaling
Flat String Formation

- Flat strings are formed after a brief period of thermal inflation: [Lyth+Stewart '95]
  - Thermal corrections trap the flat direction scalar at the origin.
  - The excess vacuum energy drives inflation until $T \sim m$.
  - When the $T \lesssim m$, the scalar rolls down the potential and oscillates.
• Number of e-foldings of thermal inflation: \( N_e \sim \frac{1}{2} \log(v^2/m^2) \).

• The oscillating scalar decays at time \( t = \Gamma^{-1} \) with
  \[
  \Gamma = \gamma \frac{m^3}{v^2}
  \]

• These decays reheat the universe to
  \[
  T_{RH} \sim g_*^{-1/4} (M_{Pl} \Gamma)^{1/2}
  \sim 100 \text{ MeV} \left( \frac{g_*}{10} \right)^{-1/4} \left( \frac{\gamma}{0.1} \right)^{1/2} \left( \frac{v}{10^{14} \text{ GeV}} \right)^{-1} \left( \frac{m}{10^3 \text{ GeV}} \right)^{3/2}
  \]

• Nucleosynthesis requires \( T_{RH} > 5 \text{ MeV} \). [Hannested '05]

• Flat-direction strings are formed at \( T \sim m \).

• Ordinary cosmic strings are formed at \( T \sim v \).
Scaling of Flat Strings

- Flat strings can zipper in addition to intercommuting.
- We study their evolution with a simple model by Tye, Wyman, and Wasserman. [TWW '05]
- They form a scaling network with many different types of string.
- Many string varieties approach an equal scaling density:

![Graph showing scaling densities for different N values](image)
The density of each string species is inversely proportional to the total number of strings types that are scaling,

\[ \rho_N \simeq \frac{1}{N_{tot}} \rho_{tot} \simeq \frac{1}{N_{tot}} G \mu (\rho_{matter} + \rho_{radiation}) \]

The flat cosmic string density is spread out among many species, each with a nearly equal equal tension.
Cosmological String Signatures
Ordinary Cosmic String Signatures

- Most cosmic string signatures are characterized by $G\mu \simeq \left( \frac{v}{\sqrt{8\pi M_{Pl}}} \right)^2$.

- Long strings:
  - String wakes can seed large scale structure.
    CMB $\Rightarrow G\mu \lesssim 10^{-7}$
    [Pogosian, Wyman, Wasserman '06, Fraisse '06]
  - Light passing by a string is gravitationally lensed. [Vilenkin '81]

- String loops:
  - Loops are not topologically stable.
  - They oscillate and emit gravity waves.
    Pulsar timing measurements $\Rightarrow G\mu \lesssim 10^{-7} - 10^{-10}$
    [DePies+Hogan '07]

- Flat direction strings have additional signatures.
String Cusps and Particle Creation

- String loops frequently form **cusps** as they oscillate.

![Cusp Diagram]

- In each cusp event, a length of string $\ell_c$ is annihilated

\[ \ell_c = \sqrt{w\ell}, \]

where $w$ is the string width and $\ell$ is the loop length.

[Blanco-Pillado+Olum ’98]

- String annihilation in cusps leads to **particle creation**. This is expected to be the dominant source of particles.

[Srednicki+Theisen ’87, Brandenberger ’87]
• The rate of energy loss from a loop to gravitational radiation is

\[ P_{\text{grav}} \approx G\mu^2 \]

• The rate of energy loss from a loop to particles by cusping is

\[ P_{\text{cusp}} \approx \mu \left( \frac{w}{\ell} \right)^{1/2} \]

where \( \ell \) is the loop length. [Blanco-Pillado+Olum '98]

• Flat strings are much wider than ordinary strings,

\[ w_{\text{flat}} \sim m^{-1} \gg v^{-1} \sim w_{\text{ordinary}}. \]

⇒ particle creation is enhanced for flat strings

• Particle creation dominates over gravitational radiation for loops smaller than

\[ \ell < \ell = \approx w \left( \frac{1}{G\mu} \right)^2. \]
Flat String Loop Signatures

- Enhanced particle creation leads to new signatures:
  - dark matter
  - modifications to nucleosynthesis and the CMB blackbody
  - cosmic rays

- The nature of the signatures depends on the typical initial loop size.

- Large initial loops, $\ell_i(t_i) \simeq 0.1 t_i$: [Ringeval et al. '05; Olum+Vanchurin '06]
  → loops lose most of their energy to gravitational radiation.

- Small initial loops, $\ell_i(t_i) \ll G \mu t_i$: [Siemens+Olum '01; Polchinski et al. '06]
  → loops lose most of their energy to particle creation

- This is a topic of current debate, even for ordinary strings.
  We therefore consider both large and small initial loop sizes.
Gravitational Wave (GW) Signatures

- The GW spectrum from flat strings is suppressed.
- This suppression is very strong for small initial loop sizes.
- For large initial loops \((\ell_i(t) = (0.1) t)\) the GW spectrum is
Particle Creation Signatures

- These are important for small initial loops ($\ell_i(t) \ll t$).
  - Dark matter:
    some of the decay products can be dark matter.
  - Nucleosynthesis:
    late decays modify light element abundances.
    $\Rightarrow G\mu \lesssim 10^{-12}$
  - CMB blackbody:
    late-time photon production modifies the CMB frequency spectrum.
    $\Rightarrow G\mu \lesssim 10^{-11}$ (COBE/FIRAS)
  - Cosmic rays: energetic decay products can make up cosmic rays.
    $\Rightarrow G\mu \lesssim 10^{-11}$ (EGRET)

- These signatures are all very enhanced due to the large width of flat-direction cosmic strings.

- (We have assumed $m = 10^3 \text{ GeV}$ and small loops for these bounds.)
Dark Matter from Flat-Direction Strings

• If the dark matter (DM) is made up of massive particles, these particles can be created as decay products from cusps.

• The amount of dark matter created in this way depends on how the string fields couple to the dark matter fields.

• For very small loops that decay entirely by cusp annihilation,

\[ \Omega_{\text{DM}}^{\text{strings}} \approx 30 \, \epsilon_1 \left( \frac{\gamma}{0.1} \right)^{1/2} \left( \frac{v}{10^{14} \text{ GeV}} \right) \left( \frac{m}{10^3 \text{ GeV}} \right)^{3/2} , \]

where \( \epsilon_1 \) is the branching fraction into DM.

• DM can also be generating during reheating after thermal inflation.
Constraints from Nucleosynthesis

- Nuclear reactions at $T \lesssim 1 \text{ MeV}$ in early universe can account for the primordial light element abundances.

- Energy released after this time can ruin this prediction.

- For very small loops that decay entirely by cusp annihilation,

$$\frac{\Delta E}{S}(t > 100 \text{ s}) \sim (10^{-12} \text{ GeV}) \left( \frac{G \mu}{2 \times 10^{-11}} \right).$$

  where $S$ is the entropy in a volume $a^3$.

- Recent estimates constrain $\Delta E/S \lesssim 10^{-12} - 10^{-14} \text{ GeV}$.

  [Kawasaki,Kohri,Moroi ’04]
Constraints from the CMB Blackbody Spectrum

- The cosmic microwave background (CMB) frequency spectrum is perfectly fit by a blackbody spectrum. [COBE/FIRAS ’96]

- Photons produced after $t_{dC}$, when double Compton scattering ceases, will distort the blackbody spectrum. [Hu+Silk ’93]

- COBE/FIRAS implies $\Delta \rho_\gamma / \rho_\gamma \lesssim 7 \times 10^{-5}$.

- For very small loops that decay entirely by cusp annihilation,

$$\frac{\Delta \rho_\gamma}{\rho_\gamma} \simeq (8 \times 10^{-5}) \left( \frac{G \mu}{2 \times 10^{-11}} \right).$$
Gravitational Lensing

- **Angular Deflection** \( \propto G\mu \).

- The tension \( \mu \) can be determined by observing several galaxies lensed by the same cosmic string. [Oguri+Takahashi '05]

- In this way we might hope to observe flat strings with *different* tensions.

- Cosmic superstrings can also form multi-tension string networks. [Tye,Wyman,Wasserman '05]

\[
\mu_N \simeq \mu_1 \left[ 1 + \frac{3}{\ln(v^2/m^2)} \right] \ln N.
\]

- In practice, it will be difficult to distinguish between these different types of cosmic strings. [Gasparini *et al.* '07]
Summary

• Cosmic strings from symmetry breaking along a supersymmetric flat direction are qualitatively different from ordinary cosmic strings.

  – Flat strings are wide: \( w \approx m^{-1} \gg v^{-1} \).

  – Higher winding modes are very stable: \( \mu_N \approx \mu_1 \left[ 1 + \frac{3}{\ln(v^2/m^2)} \right] \ln N \).

• Higher winding modes can form dynamically by zippering.

  – Zippering occurs in the early universe.

  – The scaling string network contains many string types.

• Particle creation by cusp annihilation is enhanced.

  – Gravitational wave signatures are suppressed.

  – New string signatures are possible: dark matter, CMB distortions, cosmic rays.
Extra Slides
A Simple SUSY Flat Direction

- SUSY $U(1)$ gauge theory with chiral superfields $\varphi_a$ and $\varphi_{-b}$:

$$W = \frac{\lambda}{M^{a+b-3}} \varphi_a \varphi_{-b}^b + \ldots \quad \rightarrow \quad V_F = \sum_i \left| \frac{\partial W}{\partial \varphi_i} \right|^2$$

$$V_D = \frac{g^2}{2} (a|\varphi_a|^2 - b|\varphi_{-b}|^2)^2$$

$$V_{soft} = -m_a^2 |\varphi_a|^2 - m_b^2 |\varphi_{-b}|^2 - \left( \frac{A}{M^{a+b-3}} \varphi_a \varphi_{-b}^a + h.c. \right) + \ldots$$

- The full potential is minimized with

$$\varphi_a = \sqrt{\frac{b}{a+b}} v$$

$$\varphi_{-b} = \sqrt{\frac{a}{a+b}} v$$

where

$$v \simeq \left( m M^{a+b-3} \right)^{1/(a+b-2)}, \quad m \sim A \sim \sqrt{m_i^2}.$$
• We have in mind

\[ m \sim 10^3 \text{GeV}, \quad M \sim M_{\text{Pl}} \simeq 2.4 \times 10^{18} \text{GeV}. \]

\[ \Rightarrow m \ll v \ll M \text{ for } a + b > 3 \]

• The direction \( \varphi_a = \sqrt{\frac{b}{a}} \varphi_{-b} \) is \( D\)-flat; \( V_D = 0 \).

• This direction is destabilized by the soft masses at the origin, and restabilized by the superpotential away from the origin.

• SUSY prevents quantum corrections from pushing \( m \to M \).
Initial Loop Sizes

- The spectrum of small fluctuations on cosmic strings is not fully understood, and neither is the typical initial loop size.

- In the scaling regime, the loop size is usually written as

\[ \ell_i(t) = \alpha t, \]

where \( t \) is the cosmic time.

- Estimates:

  - Standard Lore: \( \alpha \simeq G\mu \)

  - Recent Simulations: \( \alpha \approx 0.01 - 0.1 \)
    
    [Ringeval et al. '05; Olum+Vanchurin '06]

  - Recent Analytics: \( \alpha \simeq (G\mu)^{1+2\chi}, \chi > 0 \)
    
    [Siemens+Olum '01; Polchinski+Rocha '06; Dubath,Polchinski,Rocha '07]

- We therefore consider both large and small initial loop sizes.